

1. Compare “brute-force” geometry storage with the “index buffer” approach.
2. Let the drawing primitives be triangles. For rendering a grid of  $N \times N$  squares, how many vertices must be stored without an index buffer, and with an index buffer? What happens if we also account for the storage of indices, assuming that each vertex is represented only by its three coordinates and that the storage size of one coordinate is equal to the storage size of one index?
3. In the winged-edge data structure, what information is stored in an edge?
4. In the winged-edge data structure, how can we enumerate the faces adjacent to a given edge?
5. Using the winged-edge data structure, write an algorithm that enumerates all edges connected to a given vertex.
6. In the half-edge data structure, how can we traverse the faces adjacent to a given topological edge?
7. Using the half-edge data structure, write an algorithm that deletes a half-edge and updates the pointers accordingly.
8. Calculation Exercise: For the line segment obtained by linear interpolation between the points  $a = [5, 1, 4]$  and  $b = [20, -5, 10]$ , calculate the points corresponding to the parameter values  $t = 0$ ,  $t = \frac{1}{3}$ ,  $t = \frac{1}{2}$ ,  $t = \frac{2}{3}$ , and  $t = 1$ .
9. Calculation Exercise: Given the points  $p_0 = [0, 0, 0]$ ,  $p_1 = [4, 2, 1]$ ,  $p_2 = [10, 10, 10]$ , and  $p_3 = [-7, 20, -4]$ , with parameter values  $t_i = 2^i$  for  $i = 0, 1, 2, 3$ , construct the segmented line. What is the parameter domain of the segmented line? What are the coordinates of the points corresponding to  $t = 1, 2, 3, 4, 5, 6, 7, 8$ ?
10. Is it true that the segmented line passes through every control point ( $p_i$ )?
11. What is meant by  $C^0$ ,  $C^1$ , and  $C^2$  continuity?
12. Can a segmented line be  $C^1$  continuous at a junction (control point)? If yes, what is the condition?
13. Define Runge’s phenomenon.
14. What are spline curves? How do they help with the problem observed by Runge?
15. Define the problem that cubic Hermite polynomials are meant to solve, and explain how cubic Hermite polynomials are used for its solution. What do the corresponding basis polynomials look like?
16. Given three consecutive control points of a Catmull–Rom spline:  $p$ ,  $q$ , and  $r$ , what is the derivative at the middle point  $q$  if the parameter values of the points are 1, 2, and 5, respectively?
17. What does it mean for a fitted curve to interpolate the points? What does it mean for it to approximate the points?
18. Do Bézier curves interpolate or approximate their control points?
19. What is the relationship between the degree of a Bézier curve and the number of its control points?
20. Prove that  $\sum_{i=0}^n B_i^n(t) = 1$  for all  $t \in [0, 1]$  for the Bernstein polynomials. (Hint: binomial theorem)
21. \*Prove that Bézier curves satisfy affine invariance. (That is, the affine transform of the points of the curve coincides with the Bézier curve defined by the affine transforms of the control points.)
22. Do Bézier curves pass through all of their control points?
23. Given a Bézier curve with control points  $p_0 = [1, 0, 0]$ ,  $p_1 = [5, 4, 0]$ , and  $p_2 = [1, 6, 2]$ , what are the coordinates of the points on the curve corresponding to  $t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$ ?
24. How does Chaikin’s algorithm work?
25. Given the points  $p_0 = [0, 0, 0]$ ,  $p_1 = [4, 0, 0]$ ,  $p_2 = [2, 5, 2]$ , and  $p_3 = [6, 4, 4]$ , what will the control point set be after one, two, and three Chaikin corner-cutting steps?