1. What is a point, and what is a vector? What operations can be performed on points and vectors?
2. What does a right-handed and a left-handed 3D coordinate system look like?
3. Given a left-handed coordinate system with basis vectors $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$, construct a right-handed system from the same origin using $\boldsymbol{i}, \boldsymbol{j}, -\boldsymbol{k}$ (draw it!). What are the coordinates in the right-handed system of points with coordinates $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in the left-handed one?
4. Define the planar polar coordinate system. How can a point given in polar coordinates be converted to Cartesian coordinates? How can a point given in Cartesian coordinates be converted to polar coordinates?
5. What are the polar coordinates (r, φ) of the following points given in Cartesian coordinates $\begin{bmatrix} x \\ y \end{bmatrix}$?
$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}.$
Hint: use the special angles and their trigonometric values, noting that the position vectors of the points form right triangles with the axes.
6. What are the Cartesian coordinates $\begin{bmatrix} x \\ y \end{bmatrix}$ of the following points given in polar coordinates (r, φ)?
$(r, \varphi) = (1, 0)$, $(2, \frac{\pi}{2})$, $(3, \pi)$, $(4, \frac{3\pi}{2})$, $(\sqrt{2}, \frac{\pi}{4})$, $(\sqrt{8}, \frac{3\pi}{4})$, $(\sqrt{18}, \frac{5\pi}{4})$, $(\sqrt{32}, \frac{7\pi}{4})$, $(2, \frac{\pi}{3})$.
7. Define the spherical (3D polar) coordinate system. How to convert a point given in Cartesian coordinates to spherical coordinates? How to convert a point given in spherical coordinates to Cartesian coordinates?
8. What are the spherical coordinates (r, φ, θ) of the following points given in Cartesian coordinates $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$?
$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$
9. What are the Cartesian coordinates $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ of the following points given in spherical coordinates (r, φ, θ)?
$(r, \varphi, \theta) = (1, 0, 0)$, $(1, \pi, 0)$, $(1, 0, \pi)$, $(1, 0, \frac{\pi}{2})$, $(1, \frac{\pi}{2}, \frac{\pi}{2})$, $(1, 0, \frac{3\pi}{4})$, $(2, \frac{\pi}{2}, \frac{3\pi}{4})$.
10. How many points are chosen in the plane to describe the entire Euclidean plane using barycentric coordinates? What does the condition "not falling in an $n-1$-dimensional subspace" mean, and what geometric restriction does it place on the chosen points?
11. How many points are chosen in the space to describe the entire Euclidean space using barycentric coordinates? What does the condition "not falling in an $n-1$-dimensional subspace" mean, and what geometric restriction does it place on the chosen points?
12. Given the points $\boldsymbol{a} = (-1, 1)$, $\boldsymbol{b} = (2, 4)$, and $\boldsymbol{c} = (5, -2)$ in the plane, find the Cartesian coordinates corresponding to the following barycentric coordinates with respect to $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$. $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$, $(-1, 1, 1)$, $(1, -1, 1)$, $(1, 1, -1)$, $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, $(-\frac{1}{4}, \frac{1}{2}, \frac{3}{4})$.
13. Find the barycentric coordinates of the following points with respect to $a = (0, 0)$, $b = (4, 0)$, and $c = (2, 4)$. $(0, 0)$, $(4, 0)$, $(2, 4)$, $(2, 2)$, $(0, 4)$, $(8, 0)$, $(2, -4)$, $(-2, -2)$.
14. How are the Euclidean plane and space extended? What are the definitions of the projective closures of $\mathbb{E}^2$ and $\mathbb{E}^3$?
15. How are homogeneous coordinates assigned to points and vectors in Euclidean space? What do projective coordinate triplets (2D) and quadruples (3D) represent in Euclidean space, depending on the values?
16. What are the homogeneous coordinate representations of the points $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ -2 \\ -5 \end{bmatrix}$? What are the homogeneous coordinate representations of the vectors $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ -2 \\ -5 \end{bmatrix}$?

17. What are the homogeneous coordinate representations of the origin and the x, y, and z axes?

18. In Euclidean space, what elements are represented by the following homogeneous coordinates, and what are their Euclidean coordinates? $\begin{bmatrix} 6 \\ 15 \\ 9 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 8 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.