Computer Graphics

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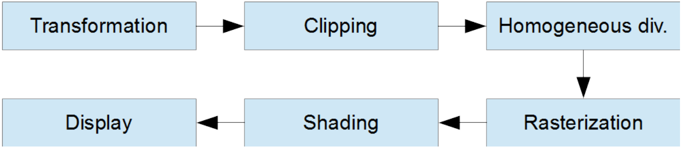
Graphics Pipeline



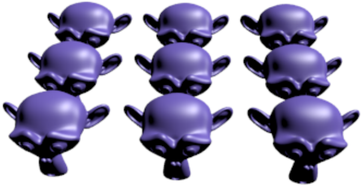
Fragment-processing

- In this step, the color of the fragments will be determined
- Now we will see how we can do this by modeling light-matter interactions
- We will also examine what texturing can be used for

Local illumination

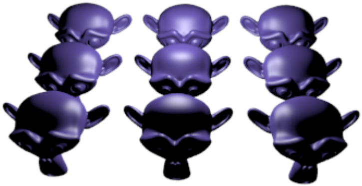
- We determine the color of the fragment according to some simplified lighting model
- For this we need
  - abstract light source models
  - abstract material models
- By combining these, we will approximate the desired lighting

Light source models



Directional light source

Light source models



Point light source

Light source models

Spot light source

Abstract light sources

- Directional light source
  - it only has direction
  - represented with one vector (in world CS)
  - the light rays are considered parallel
  - used for simulating a distant light source
- Point light source
  - it only has a position
  - represented with one point, in world CS
  - e.g.: lightbulb
- *Spot* light source
  - cone: apex position, axis direction, and a "light circle"
  - represented with a point, a vector (world CS!) and two angles
  - e.g.: table lamp

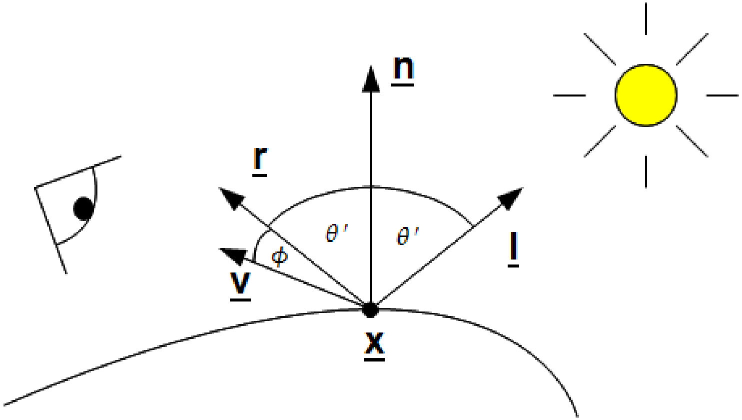
Helmholtz law

- Helmholtz reciprocity: a ray of light can be reversed
- This is good for us for two reasons:
  - It guarantees that ultimately the radiance will decrease.
  - We can look "backwards" at the rays.

Lighting models – notations

- These are used to model light-matter interactions
- $\mathbf{v} := \omega$ viewing direction
- $\mathbf{l} := \omega'$ towards the light source, vector pointing the towards the point "giving" the light
- $\mathbf{n}$ surface normal
- $\mathbf{r}$: direction of the ideal reflection from $\mathbf{l}$ incident direction, with $\mathbf{n}$ normal
- $\mathbf{v}, \mathbf{l}, \mathbf{n}$ unit vectors
- θ' is the angle between $\mathbf{l}$ and $\mathbf{n}$
- ϕ is the angle between $\mathbf{r}$ and $\mathbf{v}$

Lighting models – notations



BRDF

- Let L^{in} be the intensity of the light coming from a given direction to a point on the surface, and let L be the intensity of the light reflected from there (actually: its radiance, i.e. the power emitted from a unit surface to a unit solid angle)
- Let $\mathbf{l}$ denote the unit vector towards the light source, $\mathbf{v}$ the unit vector towards the viewpoint, and $\mathbf{n}$ the surface normal at the given point. Let θ' be the angle between $\mathbf{n}$ and $\mathbf{l}$
- Then the *bidirectional reflectance distribution function*, BRDF is

$$f_r(\mathbf{x}, \mathbf{v}, \mathbf{l}) = \frac{L}{L^{in} \cos \theta'}$$

Ideal refraction

- An ideal mirror reflects only in the specular direction $\mathbf{r}$. (see lecture 4)
- $$f_r(\mathbf{x}, \mathbf{v}, \mathbf{l}) = k_r \frac{\delta(\mathbf{r} - \mathbf{v})}{\cos \theta'}$$
- δ is the *Dirac-delta* function, which is a generalized function whose value is zero everywhere except at zero, but its integral over the real numbers is 1.
- The k_r reflection coefficient is the *Fresnel coefficient*. This depends on the refractive index of the material and its electrical conductivity.
- The *Fresnel coefficient* describes the ratio of reflected and incident energy. This physical quantity can be calculated.

Ideal refraction

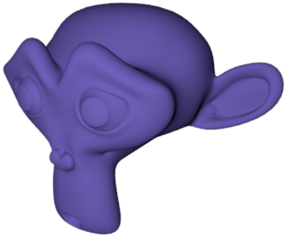
- Let $\mathbf{t}$ be the direction of ideal refraction. (see lecture 4)
- We get it similarly to the ideal reflection:
- $$f_r(\mathbf{x}, \mathbf{v}, \mathbf{l}) = k_t \frac{\delta(\mathbf{t} - \mathbf{v})}{\cos \theta'}$$
- k_t is the refraction, *Fresnel coefficient*.

Lambert's cosine law

- Good for describing optically rough, *diffuse* surfaces.
- Assumption: the amount of reflected light does not depend on the viewing direction.
- Due to Helmholtz's law, it cannot depend on the incoming direction either, i.e. it is constant:

$$f_r(\mathbf{x}, \mathbf{v}, \mathbf{l}) = k_d$$

- Only looking at this: $L_{ref} = L_i k_d \cos^+ \theta'$

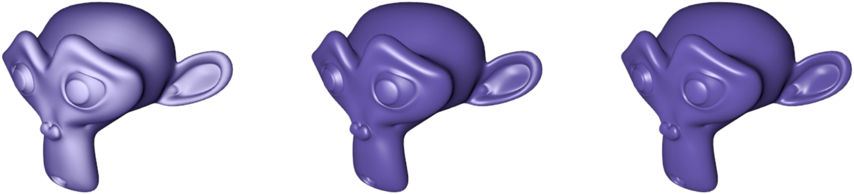


Specular reflection – Phong model

- It can used to create a "glitter", that is intensive in the specular direction but quickly fades when moving away from the specular direction
- Let ϕ be the angle between $\mathbf{r}$ specular direction and $\mathbf{v}$ view direction.
- Then $\cos\phi = \mathbf{r} \cdot \mathbf{v}$
- We are looking for a function that is large for $\phi = 0$ but quickly dies off.
- $$f_r(\mathbf{x}, \mathbf{v}, \mathbf{l}) = k_s \frac{\cos^n \phi}{\cos \theta'}$$

  Not symmetric!
- Only looking at this: $L_{ref} = L_i k_s (\cos^+ \phi)^n$

Specular reflection – Phong model



$n = 5$ $n = 25$ $n = 50$

Specular reflection – Phong-Blinn model

- Let **h** be the bisecting vector of the vectors pointing towards the light source and the viewing direction.
- $$\mathbf{h} = \frac{\mathbf{v} + \mathbf{l}}{\|\mathbf{v} + \mathbf{l}\|}$$
- Let δ be the angle between **h** and **n** normal.
- Then $\cos\delta = \mathbf{h} \cdot \mathbf{n}$
- $$f_r(\mathbf{x}, \mathbf{v}, \mathbf{l}) = k_s \frac{\cos^n \delta}{\cos \theta'}$$
- Only looking at this: $L_{ref} = L_i k_s (\cos^+ \delta)^n$

Implementation of lighting models

- The previous formulas describe the behavior of a single wavelength of light.
- We should describe the entire spectrum, but we simplify: we only use the RGB wavelengths.
- Most surfaces cannot be described by a single model, we use several models to describe the interaction between material and light.
- The amount of light from different models is *summed*.
- Since we only calculate the local illumination, we lose light from multiple reflections. We replace this separately (*ambient light*).

Ambient component

- The amount of light present everywhere in the scene.
- Formula: $k_a \cdot L_a$, where "$\cdot$" is now the multiplication per coordinate: $[a, b, c] \cdot [x, y, z] = [ax, by, cz]$
- k_a depends on the surface, let
  `vec3 ambientColor`
- Here, k_a determines what fraction of the incident light intensity is reflected by the material at the wavelengths corresponding to R, G, B. k_a is a *surface property*.
- L_a is the intensity of constant background illumination at RGB wavelengths, independent of light source and surface. L_a is a property of the *scene*.
- In *fragment shader* (in DirectX pixel shader) can be used:
  `ambientColor * ambientLight`

Lambert-law

- BRDF: $L_{ref} = L_i k_d \cos^+ \theta'$
- We called this *diffuse color*.
- k_d and L_i as before, but L_i is the current *light source property*:
  `vec3 diffuseColor`
  `vec3 diffuseLight`
- We also need: $\cos^+ \theta'$. ($\cos^+$: where cos is negative, it is 0.)
- Calculation: `clamp(dot(normal, toLight), 0, 1)`
- For this you need, `normal` = $\mathbf{n}$ and `toLight` = $\mathbf{l}$.
- In the case of a *Spot* light source, the light circle will also have to be taken into account.
- $clamp(x, a, b) = \begin{cases} a & , if\ x < a \\ x & , if\ x \in [a, b] \\ b & , if\ x > b \end{cases}$

Specular reflection – Phong model

- BRDF: $L_{ref} = L_i k_s (\cos^+ \phi)^n$, where ϕ is the angle between **r** specular direction and **v** viewing direction.
- k_d and L_i (this is a different L_i) again as before, L_i is also a the current light's *light source property*:
  ```
  vec3 specularColor
  vec3 specularLight
  ```
- n is a surface-dependent constant, let it be
  ```
  float specularPower
  ```

Specular reflection – Phong model

BRDF: $L_{ref} = L_i k_s (\cos^+ \phi)^n$; **r** specular direction and **v** viewing direction.

- We need $\cos^+ \phi$, for which we need **r** and **v**.
- **r** is the mirror image of **l** vector on **n**. Calculation:
  ```
  vec3 refl = reflect(-toLight, normal)
  ```
- **v** is the viewing direction, i.e. the unit vector pointing from the surface point to the camera.
  ```
  vec3 toEye = normalize(eyePosition - worldPos)
  ```
- Calculating $(\cos^+ \phi)^n$:
  ```
  pow(clamp(dot(refl, toEye),0,1), specularPower)
  ```

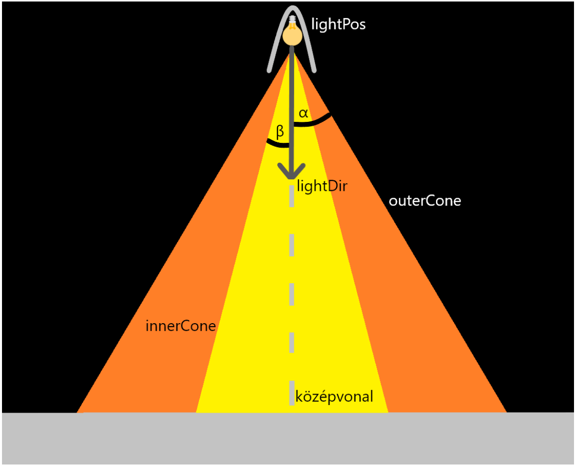
Calculating `toLight`

- Directional light source
  - Direction of the light, normalized direction vector: `vec3 lightDirection`
  - `toLight = -lightDirection`
- Point light source
  - Light's position, position vector: `vec3 lightPosition`
  - `toLight = normalize(lightPosition - worldPos)`
- *Spot* light source
  - Direction of the light, normalized direction vector: `vec3 lightDirection`
  - Light's position, position vector: `vec3 lightPosition`
  - `toLight = normalize(lightPosition - worldPos)` like a point light source

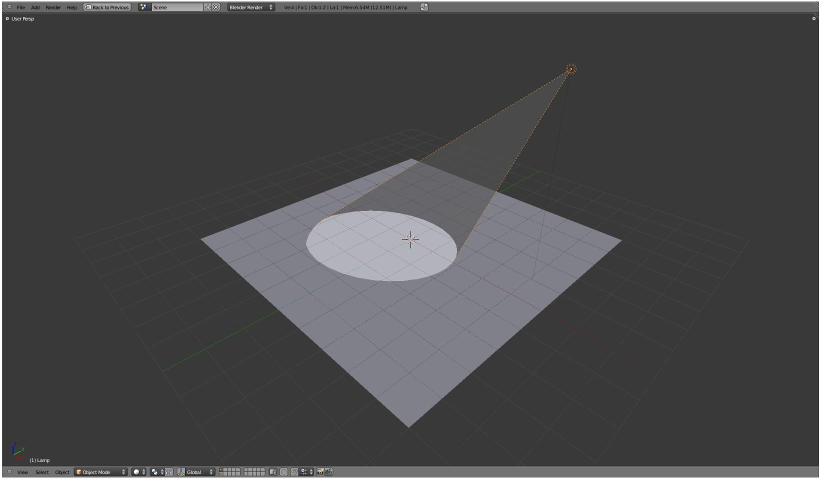
Effect of *spot* light source

- Two extra parameters:
  - inner light circle: within which it acts with full intensity
  - outer light circle: outside of which it has absolutely no effect
- Between the two, the light intensity decreases
- The light circles are considered to be (infinite) cones starting from the light source and having the same orientatoin as the direction of the light.
- A surface point is inside a cone if the line connecting the point to the position of the light source is inside the cone.

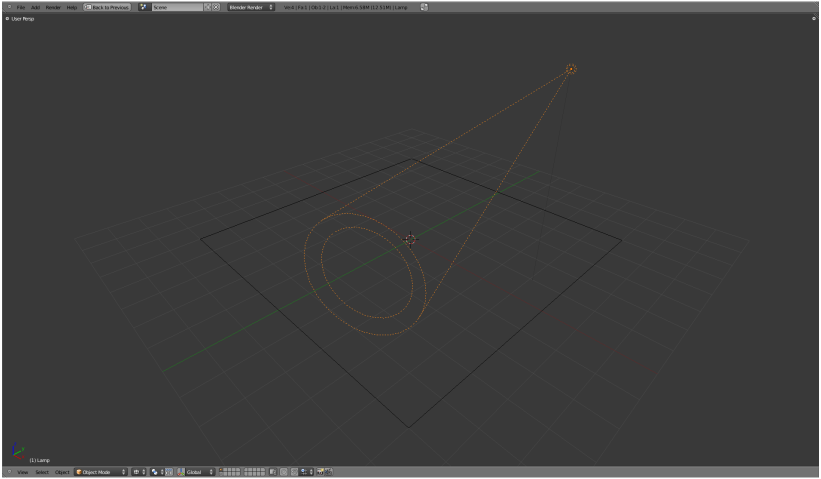
Spot light source



Spot light source



Spot light source



Effect of *spot* light source

- If the angle between `lightDirection` and `-toLight` is less than half of the opening angle of the cone, then the segment is inside the cone, i.e. the surface point is inside too
- Instead of the angles, it is sufficient to examine their cos if the max angle is $\leq 180^\circ$
- Let's represent the light circles, with cos of the half angle to the corresponding cone:
  - inner light-circle: `cosInnerCone`
  - outer light-circle: `cosOuterCone`

Effect of *spot* light source

- `smoothstep(min, max, x)`:
  - 0, if `x<min`
  - 1, if `x>max`
  - a transition between 0 and 1 (linear interpolation or cubic Hermite)
- `float spotFactor = smoothstep(cosOuterCone, cosInnerCone, dot(lightDirection, -toLight))`
- The diffuse and specular terms must be multiplied with this

In summary

Surface properties

- `ambientColor`
- `diffuseColor`
- `specularColor`
- `specularPower`
- `normal`
- `worldPos`

Light source properties

- `diffuseLight`
- `specularLight`
- `toLight`

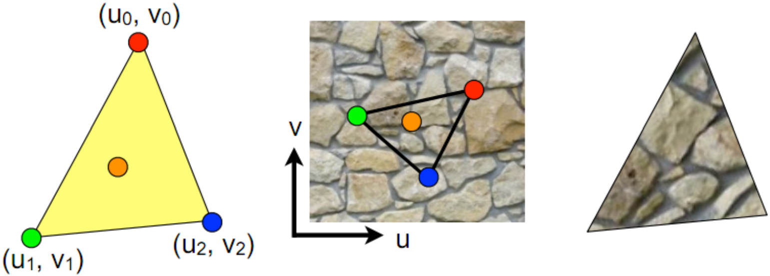
Scene properties

- `ambientLight`
- `eyePosition`

What is texturing?

- So far: *single color* material models, i.e. the entire surface is the same color.
- Not common in the real world.
- We want to give fine details.
- We need different parameters in BRDF.
- We specify these parameters – mainly color – in the *textures*.

Texturing



Texture description methods

- "Array":
  - we read from some 1/2/3 dimensional array, and its elements are *texel*s
  - Visually in 2D: we "cover/wrap" our geometry with the "image" stored in the array
  - We identify the texels with coordinates in *texture space*, the coordinates are interpreted on the $[0, 1]$ interval
- We give it as a function:
  - using some $f(x, y, z) \to$ color or $f(x, y) \to$ color function
  - this is called *procedural texturing*

Mapping methods

- We have two spaces: *image space* (screen's pixels) and *texture space* (texture's texels)
- From where to where do we map?
- Texture space $\rightarrow$ image space: *texture based mapping*
  - » For each *texel*, we search for its corresponding *pixel*.
  - $\oplus$ Efficient.
  - $\ominus$ It is not guaranteed that we "hit" every pixel, if we hit a pixel we might hit it multiple times.
- Image space $\rightarrow$ texture space: *image space based mapping*
  - » For each *pixel*, we search for its corresponding *texel*.
  - $\oplus$ It fits well with incremental image synthesis.
  - $\ominus$ It requires the inverse of the parameterization and projection transformations.

Parameterization

- How can we decide for each surface point, which elements of the array are needed or with which parameters to evaluate the procedural texture function?
- *texture coordinates* are assigned to each point of the surface.
- The assignment of texture coordinates to the surface is now called *parameterization*.
- In the following, we will talk about 2D textures.

Parameterizing parametric surfaces

- Parametric surface:

$$F \in \mathbb{R}^2 \to \mathbb{R}^3, \quad F(u, v) := (x, y, z)$$

- naturally the u, v parameters can be used as texture coordinates.
- If $\mathcal{D}_F \neq \mathcal{D}_{tex}$, then we need to transform u, v.
- E.g.:
  - Cylindrical surface
  - $\mathcal{D}_F = [0, 2\pi] \times [0, h]$, $F(u, v) := (\cos u, v, \sin u)$
  - Texture space: $(\bar{u}, \bar{v}) \in [0, 1] \times [0, 1]$
  - Transformation: $\bar{u} := u/2\pi$, $\bar{v} := v/h$

Parameterizing triangles

- Let triangle vertices: $p_i = (x_i, y_i, z_i) \in \mathbb{R}^3$, $i \in \{1, 2, 3\}$, as well as their corresponding vertices in texture space: $t_i = (u_i, v_i) \in \mathbb{R}^2$, $i \in \{1, 2, 3\}$.
- We are looking for a mapping $\mathbb{R}^3 \to \mathbb{R}^2$ such that $p_i \mapsto t_i$ and it maps a triangle to triangle.
- The simplest such mapping is the linear mapping, which can be given by a 3×3 matrix.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{bmatrix} \cdot \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{P} \cdot \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Parameterizing triangles

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{bmatrix} \cdot \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{P} \cdot \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

- Nine unknowns, nine equations
- Texture based mappig!
- For screen based mapping we need the inverse transformation:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{P}^{-1} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Texturing and ray tracing

- Surface-ray intersection: in world coordinate system
- Inverse transformations:
  - World CS $\rightarrow$ Model CS
  - Model CS $\rightarrow$ texture space

Model CS $\rightarrow$ texture space

- Parametric surface
  We get u, v during intersection calculations, it does not need to be calculated separately.
- Triangles
  The previously derived $\mathbf{P}^{-1}$ required.
  Speed up option: if neither the triangle itself nor the texture coordinates change in the vertices, then $\mathbf{P}^{-1}$ is constant.

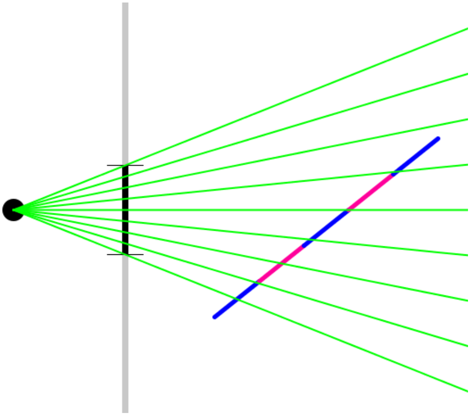
Parameterizing triangles

- In practice, this can be calculated quickly with an incremental algorithm.
- If the texture coordinates are given in the three vertices, they can be calculated for each pixel with the algorithm used for filling the triangles.
- Let α, β, γ be the barycentric coordinates of the points on the surface where $\mathbf{p} = \alpha \mathbf{p}_1 + \beta \mathbf{p}_2 + \gamma \mathbf{p}_3$
- Then we get the texture coordinate for $\mathbf{p}$ with $t = \alpha t_1 + \beta t_2 + \gamma t_3$

Perspective-correct texture mapping

- The linear interpolation of texture coordinates will give the wrong image, if we are not using only affine transformations on the triangles.
- I.e: 99% of the time.

Perspective-correct texturing



Perspective-correct texturing

- So linear interpolation of u, v won't be good for non-affine transformations – because they are not linear in screen space
- After we transform, but before homogeneous divide, our coordinates should be $[x_t, y_t, z_t, w_t]$
- After homogeneous divide: $[x_s, y_s, z_s, 1] = [\frac{x_t}{w_t}, \frac{y_t}{w_t}, \frac{z_t}{w_t}, 1]$
- If we interpolate the texture coordinates based on $[x_s, y_s, z_s, 1]$, then it won't be correct.
- In contrast to that: if we interpolate $\frac{1}{w}$, it will remain correct! Moreover, any $\frac{q}{w}$ value is well interpolated!
- Instead of α, β, γ we use the coordinates $\alpha_w, \beta_w, \gamma_w$ in world CS, and
- interpolating $\frac{\alpha_w}{w}, \frac{\beta_w}{w}, \frac{\gamma_w}{w}$, we get the correct texturing.

Perspective-correct texturing - proof 1/3

- Now we denote the screen coordinates as X, Y and the spatial coordinates as x, y, z, where the relation between the two:

$$X = \frac{x}{z}\ ,\ \ Y = \frac{y}{z}$$

from which obviously $x = Xz\ ,\ \ y = Yz$.

- Let us assume that the point above is a vertex of a triangle (or polygon in a general case) and let the implicit equation of this primitive's plane be

$$Ax + By + Cz = D$$

- Let u, v denote the texture coordinates of the spatial primitive points, which we assume can be expressed as a linear function of the x, y, z coordinates:

$$u = ax + by + cz + d\ ,\quad v = ex + fy + gz + h$$

Perspective-correct texturing - proof 2/3

Let's show that $1/z$ is a linear function of X, Y:

- Let's start from the implicit equation of our primitive's plane:

$$Ax + By + Cz = D$$

  when we substitute x and y with the screen coordinates (X, Y) we get the following expression:

$$AXz + BYz + Cz = D$$

- Divide this by zD (i.e. express $1/z$):

$$\frac{1}{z} = \frac{A}{D}X + \frac{B}{D}Y + \frac{C}{D}$$

- Since A, B, C, D are constants (i.e. independent of X, Y), the above expression is indeed the $1/z$ as a linear function of X, Y

Perspective-correct texturing - proof 3/3

Let's show that $u/z, v/z$ is a linear function of X, Y:

- We assumed that texture coordinates are linear in spatial coordinates, i.e

$$u = ax + by + cz + d$$
$$v = ex + fy + gz + h$$

by substituting the spatial coordinates with the screen coordinates, we get this expression

$$u = aXz + bYz + cz + d$$
$$v = eXz + fYz + gz + h$$

Perspective-correct texturing - proof 3/3

Let's show that $u/z, v/z$ is a linear function of X, Y:

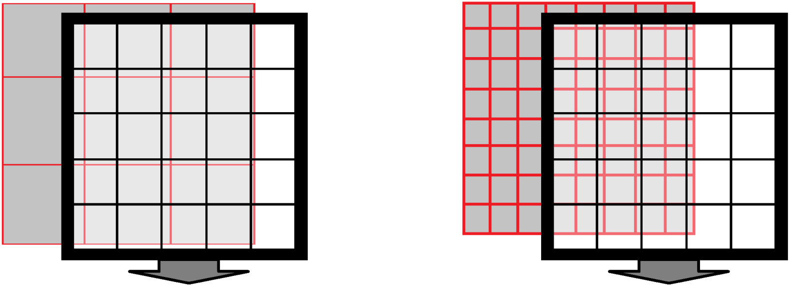
- Divide by z and substitute the $1/z$ on the right side with the previously obtained expression:

$$\frac{u}{z} = aX + bY + c + d\left(\frac{A}{D}X + \frac{B}{D}Y + \frac{C}{D}\right)$$
$$\frac{v}{z} = eX + fY + g + h\left(\frac{A}{D}X + \frac{B}{D}Y + \frac{C}{D}\right)$$

and they are indeed linear, because by rearranging the above

$$\frac{u}{z} = \left(a + d\frac{A}{D}\right)X + \left(b + d\frac{B}{D}\right)Y + \left(c + d\frac{C}{D}\right)$$
$$\frac{v}{z} = \left(e + h\frac{A}{D}\right)X + \left(f + h\frac{B}{D}\right)Y + \left(g + h\frac{C}{D}\right)$$

Texture filtering



- It is rare to have exactly one texel per pixel.
- Magnification: the pixel size is smaller than the texel size – multiple pixel per texel. OpenGL: `GL_TEXTURE_MAG_FILTER`
- Minification: the pixel size is larger than the texel size – multiple texel per pixel. OpenGL: `GL_TEXTURE_MIN_FILTER`
- Another problem: if something is linear in texture space, then it's not linear in screen space because of the perspective

Magnification

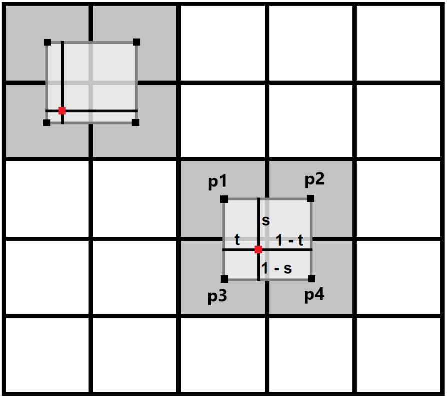
The pixel size is smaller than the texel size – multiple pixel per texel

- Without filtering: we use the value of the texel closest to the pixel's center
- Bilinear filtering: we take the weighted average of the nearest four texels.

Bilinear filtering

$$\mathbf{b}(s, t) = (1 - t)((1 - s)\mathbf{p}1 + s\mathbf{p}3) + t((1 - s)\mathbf{p}2 + s\mathbf{p}4)$$

$s, t \in [0, 1]$



Minifiacation

The pixel size is larger than the texel size – multiple texel per pixel

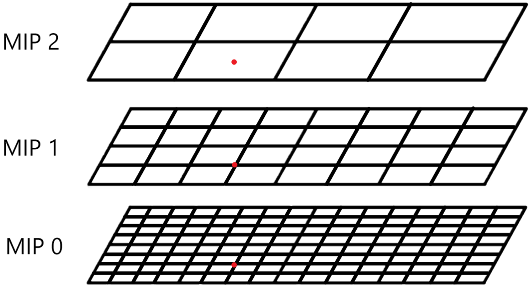
- The accurate sampling would be: transform the square of the pixel into texture space and take the average of the selected texels.
- Instead: we also take a square in the texture space.
- In practice: averaging of an arbitrary square of the texture is too resource demanding, instead we should either use fewer texels or *MIP maps*
- Fewer pixel:
  - Without filtering: we use the value of the texel closest to the pixel's center
  - Bilinear filtering: we take the weighted average of the nearest four texels.

MIP-maps

- MIP: *multum in parvo* – much in a small space
- We generate a "pyramid" from the texture, halving its size at each level

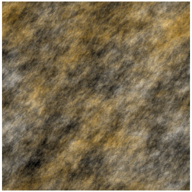
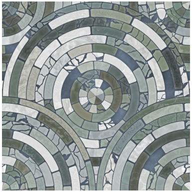
MIP-maps

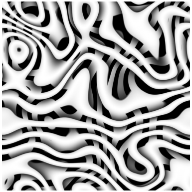
- During filtering, we select the appropriate level based on the ratio of pixel/texel area and read from there.
- Even within the given MIP-map, reading can be unfiltered or bilinearly filtered.
- *Trilinear filtering*: we use use two adjacent levels, we use bilinear filtering within the levels, then we take the weighted average of the results



Procedural textures

- The textures can be specified with a function instead of an "array".
- Texture coordinates: the function parameters.



generated with Filter Forge

Properties

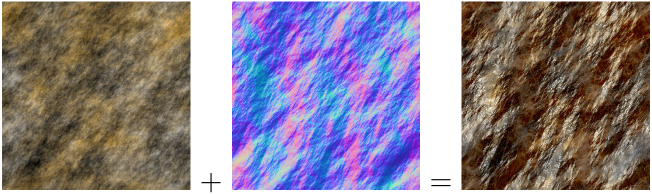
- Advantages:
  - A lot less storage space is required
  - Any resolution – the only limit is numerical accuracy
  - $\Rightarrow$ In case of magnification we don't need to filter.
    (Unfortunately, it does not solve the minification problem)
- Drawbacks:
  - High computational demand.
  - It can be harder to modify it.

Non-color textures

- Textures can be used to describe any property of a surface point.
- These properties can be for example:
  - surface normal – *bump and normal mapping*
  - displacement – Displacement *mapping*
  - light source visibility – *Shadowmaps*
  - mirror like reflection – *Reflection mapping/Environment mapping*

Bump or normal mapping

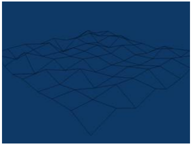
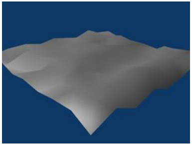
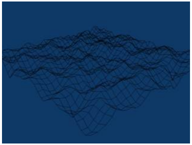
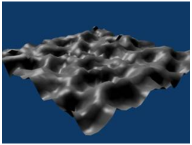
- With the texture we specify normal vectors.
- Instead of the original normals of the surface, we use them when calculating the lighting.
- It gives the appearance of a bumpy/rough surface until you look at it, at a very flat angle.



generated with Filter Forge

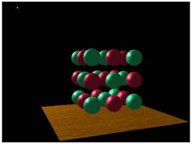
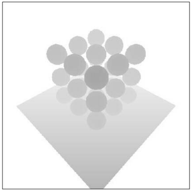
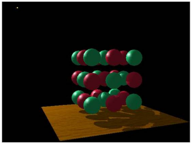
Displacement mapping

- The surface point is actually moved
- We can only move the vertices of triangles $\Rightarrow$ depends on the resolution of the geometry.
- Even at a flat angle, we get a good result.

Shadowmaps

- From the light source's point of view, we create a texture in which the distances from the light source are stored.
- During the actual drawing, based on the shadowmap, we decide for each point whether it is directly affected by the light or not.

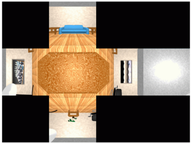

+

=


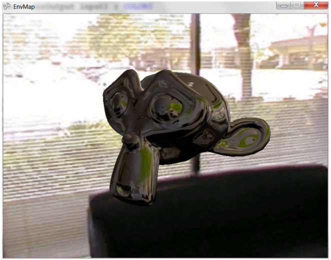
Reflection mapping

- Can be used for flat mirrors.
- We create a separate image, saved in a texture, of what is visible in the reflected direction.
- This will be applied to the surface as a texture during the final drawing.
- It only gives a single bounce/reflection.
- It must be done separately for each mirror.

Environment mapping

- We consider the environment of our scene to be infinitely distant, in the query only the direction matters, not the position.
- We store it in a special texture. (Typically *Cubemap*.)
- When drawing, for reflective surfaces, we read from this according to the direction of the reflection.

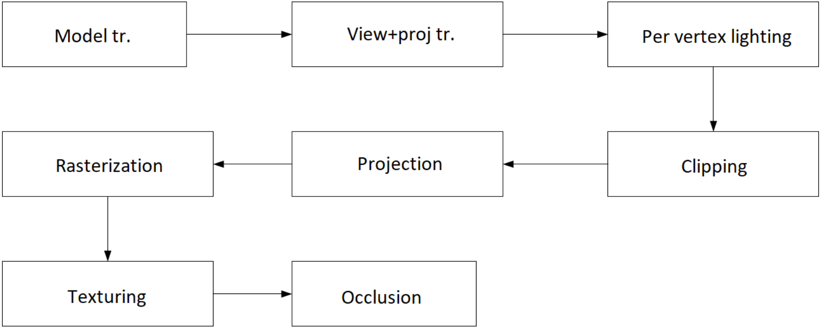




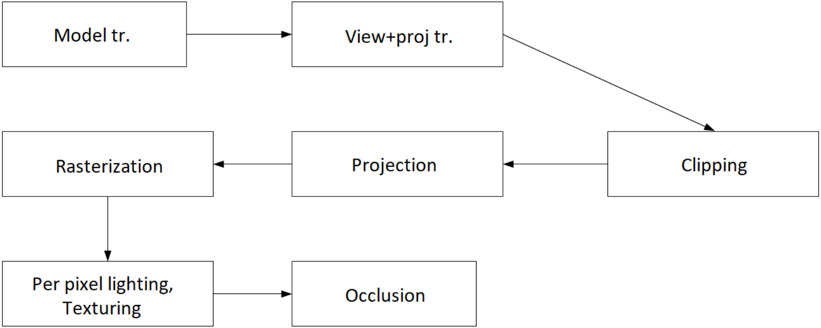
In summary

- All surface-optical properties can be specified with a constant or even with a texture.
- (Even more if we are clever!)
- All vectors and points are given in the world coordinate system.
- `eyePosition` must be updated when the view changes!
- Problem: our model is in model CS, and the *Vertex Shader* will transfer it to normalized device CS!
- Solution: let's also calculate the world coordinates with the *Vertex Shader*, and pass the interpolated value to the *Fragment Shader*!

Incremental image synthesis



Incremental image synthesis



Pipeline

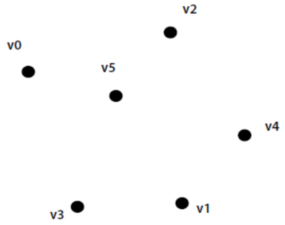
- We divide the work into subtasks
- Each subtask is processed by a different processing unit (processor) (ideally)
- The input of each unit is the output of the unit preceding it in the pipeline

Pipeline

- In order for a unit to start working, it does not have to wait for work on the *entire* workpiece to be completed, only the parts in front of it need to be ready
- $\rightarrow$ if we introduce n pipeline **stage**, then at a given moment we can work on up to n elements at the same time (after the initial start-up)
- Not just GPU! Even the Pentium IV had 20 pipeline stages (a GeForce 3 has 6-800)

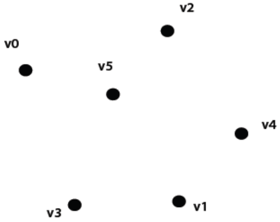
Parallelizations in practice – per vertex

- When we call glDrawArrays, glDrawElements etc., then the pipeline starts on the vertices of the primitive
- We transform each **vertex** *independently*, *in parallel*

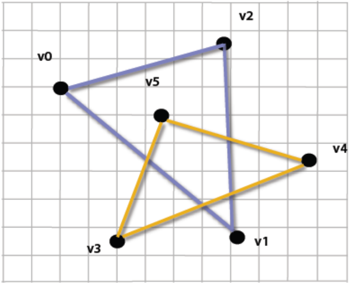


Parallelizations in practice – per primitive

- All **primitives** to be drawn are clipped *independently*, *in parallel*



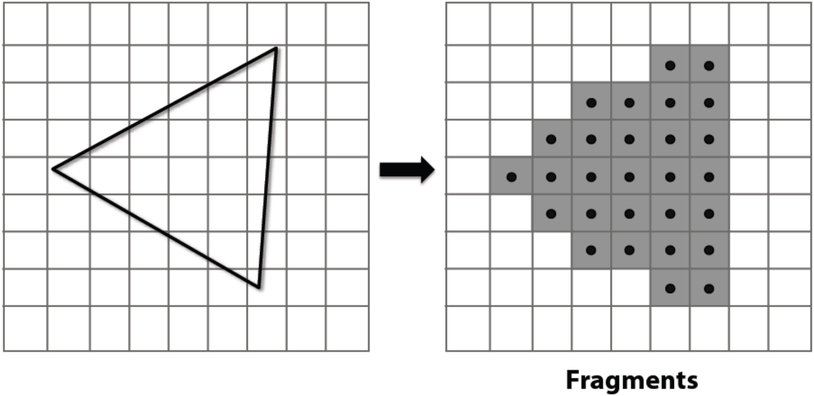
Vertices



Primitives (triangles)

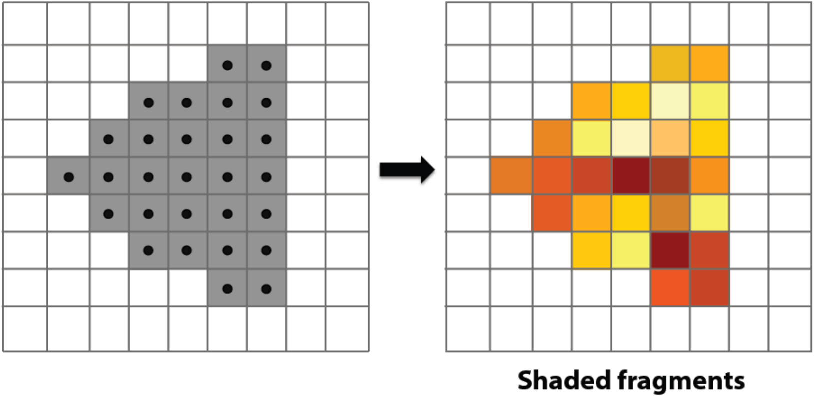
Parallelizations in practice – per primitive

- All **primitives** to be drawn are rasterized *independently*, *in parallel*



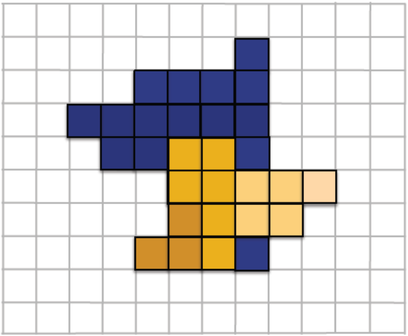
Parallelizations in practice – per fragment

- All **fragments** to be drawn are colored *independently*, *in parallel*



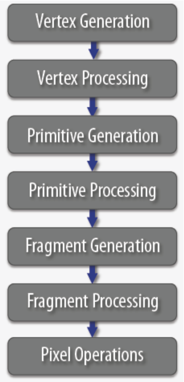
Parallelizations in practice – per pixel

- For each **pixel** corresponding to a fragment, we define the color of the pixel displayed on the screen (+visibility with z-buffer)



Pixels

Pipeline on hardware



GPU programming

- No explicit parallelization
- Because we get that from working on different elements independently, in parallel (even in time)

Compiling shaders

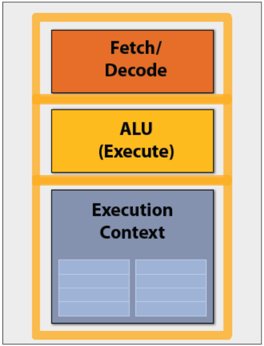
```
sampler mySamp;
Texture2D<float3> myTex;
float3 lightDir;

float4 diffuseShader(float3 norm, float2 uv)
{
  float3 kd;
  kd = myTex.Sample(mySamp, uv);
  kd *= clamp( dot(lightDir, norm), 0.0, 1.0);
  return float4(kd, 1.0);
}
```

1 input fragment record

```
<diffuseShader>:
sample r0, v4, t0, s0
mul  r3, v0, cb0[0]
madd r3, v1, cb0[1], r3
madd r3, v2, cb0[2], r3
clmp r3, r3, l(0.0), l(1.0)
mul  o0, r0, r3
mul  o1, r1, r3
mul  o2, r2, r3
mov  o3, l(1.0)
```

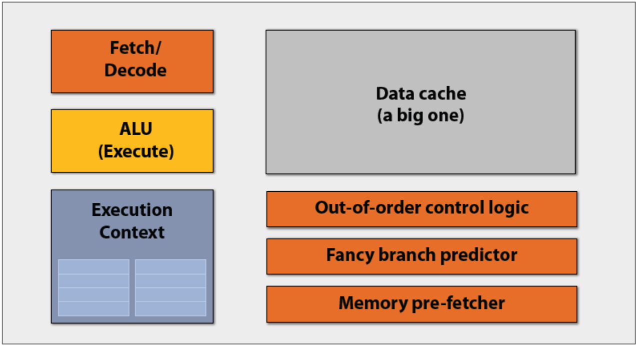
1 output fragment record

Executing shaders



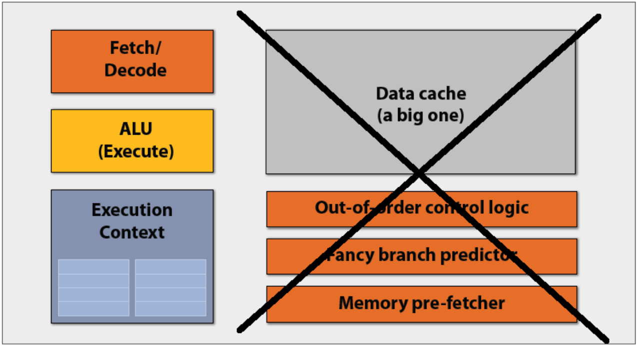

```
<diffuseShader>:
sample r0, v4, t0, s0
mul  r3, v0, cb0[0]
madd r3, v1, cb0[1], r3
madd r3, v2, cb0[2], r3
clmp r3, r3, l(0.0), l(1.0)
mul  o0, r0, r3
mul  o1, r1, r3
mul  o2, r2, r3
mov  o3, l(1.0)
```

A typical CPU

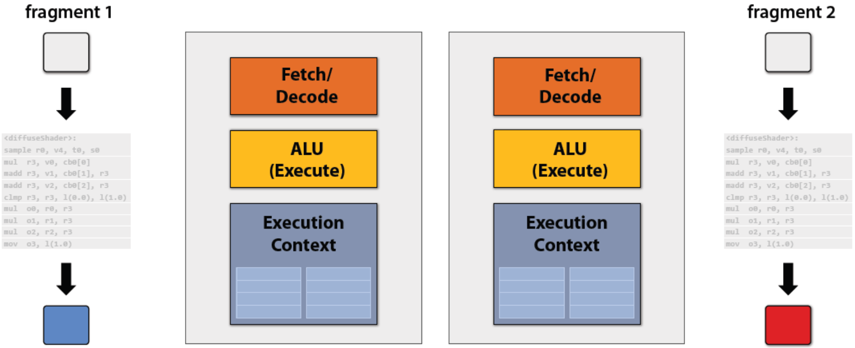


Processing unit of a GPU

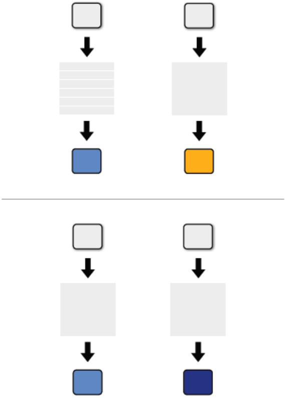
Remove the components that help with fast execution of only one thread

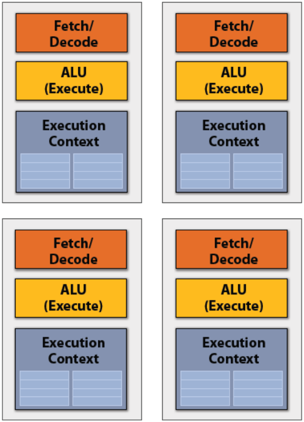


GPU – 2 processing unit, 2 shader



GPU – 4 processing unit, 4 shader

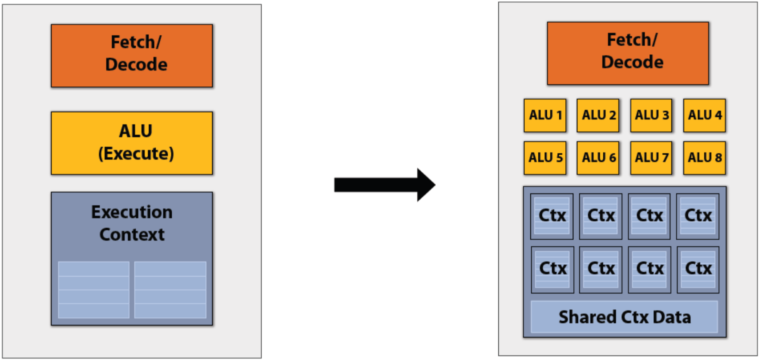




GPU processing units

- In the previous figures, each processing unit ran a different shader
- But this is not necessary!
- A triangle can be made into many fragments $\rightarrow$ the same shader must be run for each!
- $\Rightarrow$ we reduce costs by assigning a common instruction fetcher to several ALUs (i.e they execute the same program) $\rightarrow$ **S**ingle **I**nstruction **M**ultiple **D**ata

GPU processing unit

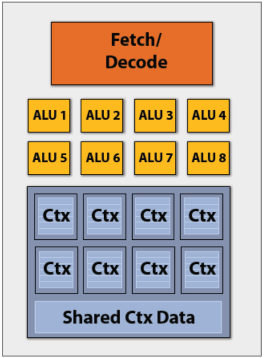


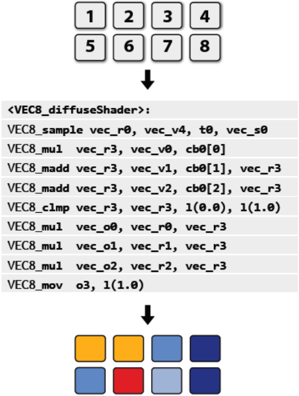
GPU processing unit

```
<diffuseShader>:
sample r0, v4, t0, s0
mul  r3, v0, cb0[0]
madd r3, v1, cb0[1], r3
madd r3, v2, cb0[2], r3
clmp r3, r3, l(0.0), l(1.0)
mul  o0, r0, r3
mul  o1, r1, r3
mul  o2, r2, r3
mov  o3, l(1.0)
```

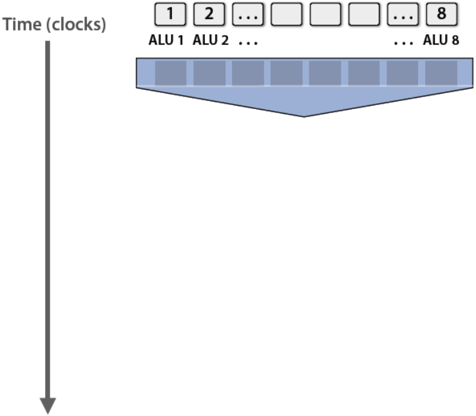
```
<VEC8_diffuseShader>:
VEC8_sample vec_r0, vec_v4, t0, vec_s0
VEC8_mul  vec_r3, vec_v0, cb0[0]
VEC8_madd vec_r3, vec_v1, cb0[1], vec_r3
VEC8_madd vec_r3, vec_v2, cb0[2], vec_r3
VEC8_clmp vec_r3, vec_r3, l(0.0), l(1.0)
VEC8_mul  vec_o0, vec_r0, vec_r3
VEC8_mul  vec_o1, vec_r1, vec_r3
VEC8_mul  vec_o2, vec_r2, vec_r3
VEC8_mov  o3, l(1.0)
```

GPU processing unit



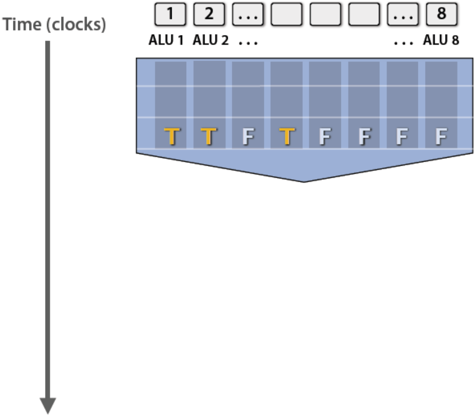


Branching




```
<unconditional
 shader code>

if (x > 0) {
    y = pow(x, exp);
    y *= Ks;
    refl = y + Ka;
} else {
    x = 0;
    refl = Ka;
}

<resume unconditional
 shader code>
```

Branching




```
<unconditional
 shader code>

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    y = pow(x, exp);
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<resume unconditional
 shader code>
```
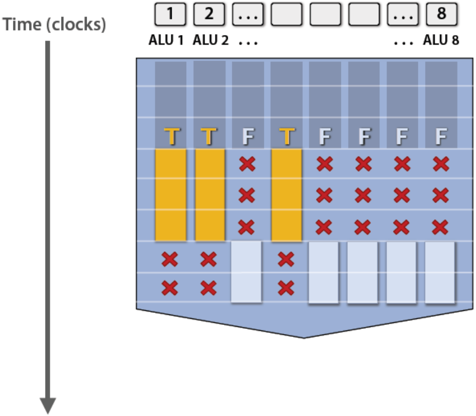

Branching




```
<unconditional
 shader code>

if (x > 0) {
    y = pow(x, exp);
    y *= Ks;
    refl = y + Ka;
} else {
    x = 0;
    refl = Ka;
}

<resume unconditional
 shader code>
```

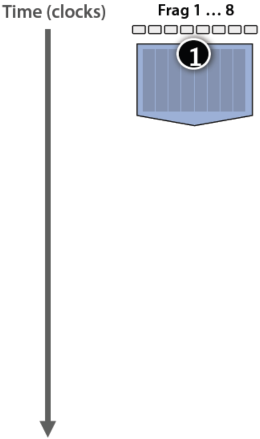
Branching

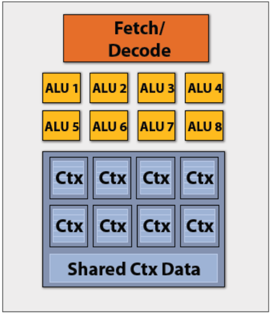
- The ALU1-ALU8 denotes the ALUs with shared instruction fetcher
- T appeared for those ALUs whose data set the branching condition to *true*
- And F for those with *false*
- The instructions of both branches (!) must be evaluated
- In the worst case, the performance in the figure above drops to one eighth!
- Today's GPUs have 16-64 ALUs on a shared instruction stream

Stalls

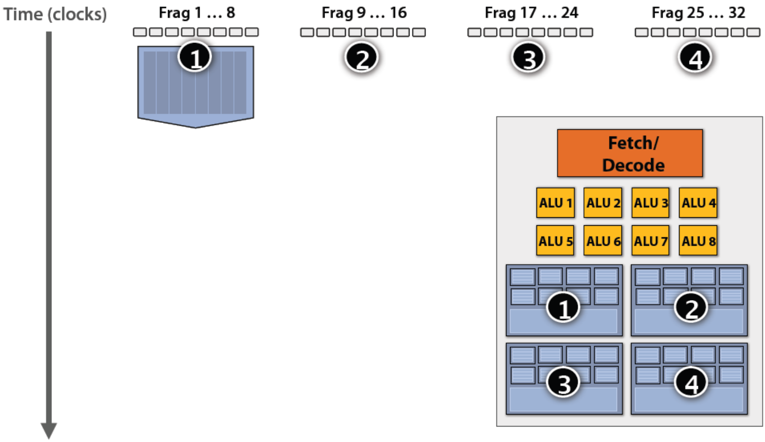
- On one ALU with multi-threading we process multiple fragment/vertex/etc. $\rightarrow$ storing multiple execution contexts (code, temporal data, etc.)
- Because some operations are much slower than others
- For example: a texture query in a shader is 100x or even 1000x slower than an arithmetic (vector) operation!

Paging

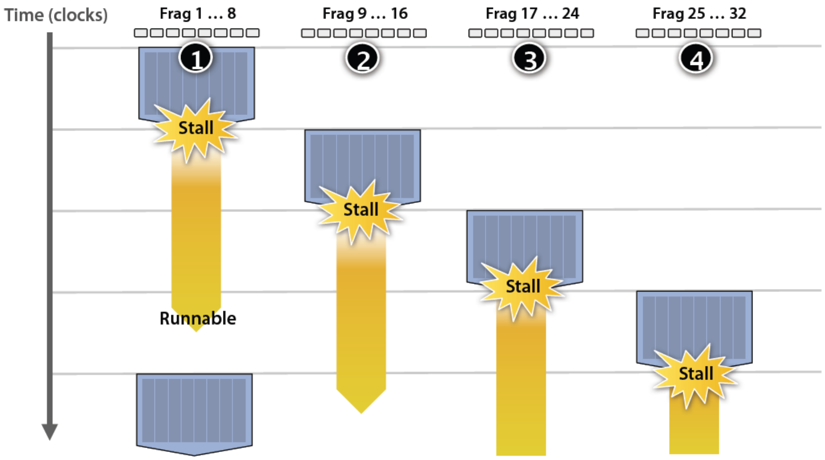

Time (clocks)
Frag 1 … 8
1


Fetch/
Decode
ALU 1
ALU 2
ALU 3
ALU 4
ALU 5
ALU 6
ALU 7
ALU 8
Ctx
Ctx
Ctx
Ctx
Ctx
Ctx
Ctx
Ctx
Shared Ctx Data

Paging


Time (clocks)
Frag 1 … 8
Frag 9 … 16
Frag 17 … 24
Frag 25 … 32
1
2
3
4
Fetch/
Decode
ALU 1
ALU 2
ALU 3
ALU 4
ALU 5
ALU 6
ALU 7
ALU 8
1
2
3
4

Paging


Time (clocks)
Frag 1 … 8
Frag 9 … 16
Frag 17 … 24
Frag 25 … 32
1
2
3
4
Stall
Stall
Stall
Stall
Runnable

GPU processing unit

The following observations played an important role in the construction of GPU stream processors:

1. Remove the components that help with fast execution of only one thread
2. $\Rightarrow$ we reduce costs by assigning a shared instruction fetcher to several ALUs (i.e they execute the same program) $\rightarrow$ **S**ingle **I**nstruction **M**ultiple **D**ata
3. On one ALU with multi-threading we process multiple fragment/vertex/etc. $\rightarrow$ storing multiple execution contexts (code, temporal data, etc.)